

Comment: Use of Piecewise Regression Models to Estimate Changing Relationships in Fisheries

Maceina (2007) recently proposed using piecewise regression to estimate size- and age-related mortality rates in fish. Piecewise regression (also called change-of-phase, segmented, hockey stick, and joinpoint regression) has also been used to fit stock–recruitment and individual growth curves for fish (Zweifel and Lasker 1976; Laidig et al. 1991; Barrowman and Myers 2000), to identify critical thresholds for environmental features (Toms and Lesperance 2003; Homan et al. 2004; Denoël and Ficetola 2007), and to analyze trends in cancer rate incidence (Kim et al. 2000; Tiwari et al. 2005). The use of piecewise regression to estimate differential mortality from age- and length-related models is an interesting application of this technique and one in which many fishery managers and researchers will probably be interested. One clear advantage of using piecewise regression to fit segmented catch curves is that the imposed continuity restriction among segments avoids the jumps and falls that can result when separate linear regression models are fit to different ages of a fish sample (Seber and Wild 2003).

While we agree with Maceina (2007) that piecewise regression is a useful tool, we believe it is important to acknowledge that fitting piecewise regression models can be challenging. Standard methods are prone to yield different parameter estimates depending on the assigned starting values (Lerman 1980; Seber and Wild 2003; Toms and Lesperance 2003). This sensitivity to starting values can be demonstrated using the fish age and \log_e transformed number-at-age data provided in Maceina (2007). In the SAS (SAS Institute 2003) PROC NLIN code provided in the appendix of Maceina (2007), a starting value of 10 is specified, which results in an estimated knot (joinpoint) of 10.3. If the starting value is changed to 11, however, the estimated knot becomes 11.2 and the estimated instantaneous total mortality becomes 0.4907 for fish of ages 3–11 and 0.1664 for fish of age 11 and above. The sum of squared errors for the estimated model with a knot of 10.3 is 5.53, while that for a model with a knot of 11.2 is 5.59. Thus, the piecewise regression model converges to a local rather than a global least-squares minimum with a starting value of 11. While the differences in parameter estimates that result from these different starting values are not large, the fish age and

\log_e transformed number-at-age data presented in Maceina (2007) constitute a rather “clean” data set. From our experience, differences in parameter estimates can be substantially larger with “noisier” data sets.

Part of the difficulty in fitting piecewise regression models stems from the fact that the models are nonsmooth functions with discontinuous first derivatives (Lerman 1980; Seber and Wild 2003), which can affect the performance of optimization algorithms. However, even when one uses optimization algorithms that do not require continuous first derivatives, there is no guarantee that a model will converge to a global minimum because local minima often exist for piecewise regression problems (Lerman 1980). To illustrate, we used PROC NLP in SAS (SAS Institute 2007) to fit a piecewise regression model to the fish age and \log_e transformed number-at-age data provided in Maceina (2007) by least squares using the Nelder–Mead simplex optimization algorithm, which is widely recommended for nonsmooth functions (Seber and Wild 2003). We obtained the same two sets of parameter estimates with starting values of 10 and 11 that we obtained with PROC NLIN.

Because of the sensitivity of piecewise regression models to the assigned starting values, we do not recommend using single starting values for model parameters. Rather, we suggest using a grid-search method to fit these models (Lerman 1980). Alternatively, Hudson (1966) and Seber and Wild (2003) describe multistage approaches for fitting piecewise regression models. Other methods that have been suggested for fitting piecewise regression models include using median functions (Shuai et al. 2003) or approximating the segmented model with a model with continuous first derivatives (Seber and Wild 2003; Toms and Lesperance 2003), but the grid-search method appears to be the one of the more widely used for these models (Kim et al. 2000; Piepho and Ogutu 2003). Implementing a grid search for estimating parameters is straightforward in SAS, although we caution that it may be necessary to use a relatively fine grid for all model parameters (not just the knot) to ensure that the global least-squares minimum is found. Grid-search estimation is also used in Joinpoint 3.0 (National Cancer Institute 2005), which is a freeware

computer program for fitting piecewise regression models that can be downloaded from the World Wide Web. Using Joinpoint 3.0, we successfully obtained the parameter estimates that globally minimized the sum of squares for a two-segment catch curve for the fish age and \log_e transformed number-at-age data provided in Maceina (2007).

As noted above, reparameterization of the model so that it is approximated by a smooth function (followed by standard nonlinear regression) has been suggested as an alternative to the grid-search method for fitting piecewise regression models (Seber and Wild 2003). We caution that this approach does not address the issue of local minima in the sum-of-squares surface. For the particular application of Maceina (2007), when we reparameterized the model to be a smooth function we still encountered problems with convergence to the same different solutions when using starting knot values of 10 and 11. Accordingly, we strongly encourage analysts to try a range of initial knot values when doing piecewise regressions, even when alternative fitting approaches are adopted.

Another important issue analysts face when considering piecewise models is how to decide whether the segmented function is a better choice than a single (linear in the catch-curve application) segment. Maceina (2007) reports an apparently unadjusted coefficient of determination (R^2) for the linear and segmented catch curves but otherwise does not report a quantitative basis for selecting the segmented function. The linear model is a special case of the segmented model in which the values for the two slopes are identical, but it is fit with two parameters rather than the four required by the segmented model. Thus, the segmented model will always produce an unadjusted R^2 as high as or higher than that of the linear catch curve. Such an R^2 value does not mean that there really are two segments or even that a two-segment model is a better predictor than a linear model of other data from the population from which the sample was drawn. The segmented model could be overfit to the sample data and thus produce poorer predictions (see section 1.4 of Burnham and Anderson 2002). Furthermore, even an adjusted R^2 , though useful as a descriptive statistic, appears to perform poorly when used for model selection (McQuarrie and Tsai 1998, as cited in Burnham and Anderson 2002). Thus, we do not think that an improvement in R^2 should be used as a basis for choosing between models.

Hypothesis tests and information-theoretic approaches are both widely used objective methods for choosing among alternative models, and there is an extensive literature on the relative merits of these approaches in different situations (e.g., Burnham and Anderson 2002;

Seber and Wild 2003). For piecewise regression models, both permutation tests for a difference in slopes between segments (Kim et al. 2000) and the Bayes information criteria (BIC; Tiwari et al. 2005) have been suggested as objective measures for choosing among linear and segmented models. Permutation tests are often preferred as a hypothesis testing method for piecewise regression problems because the nondifferentiability of the model and the existence of local minima in the sum-of-squares surface poses problems for some other approaches. We do not know of any reason why the BIC should be preferred to other information criteria (such as Akaike's information criteria) in piecewise regression applications. Although these specific model selection approaches could be programmed in SAS, they are also available in Joinpoint 3.0. Both the permutation (with $\alpha = 0.05$) and BIC model selection approaches selected the segmented catch curve over the linear catch curve when fit to the fish age and \log_e transformed number-at-age data in Maceina (2007).

In summary, we anticipate that fishery managers and researchers will be greatly interested in using piecewise regression and that the description provided by Maceina (2007) will facilitate the application of this technique. Our primary purpose here is to sound a note of caution regarding the difficulties associated with fitting these types of models and to point out some alternative approaches for choosing between segmented and linear catch curves.

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References

- Barrowman, N. J., and R. A. Myers. 2000. Still more spawner–recruitment curves: the hockey stick and its generalizations. *Canadian Journal of Fisheries and Aquatic Sciences* 57:665–676.
- Burnham, K. P., and D. R. Anderson. 2002. Model selection and multimodel inference: a practical information-

- theoretic approach, 2nd edition. Springer-Verlag, New York.
- Denoël, M., and G. F. Ficetola. 2007. Landscape-level thresholds and newt conservation. *Ecological Applications* 17:302–309.
- Homan, R. N., B. S. Windmiller, and J. M. Reed. 2004. Critical thresholds associated with habitat loss for two vernal pool-breeding amphibians. *Ecological Applications* 14:1547–1553.
- Hudson, D. J. 1966. Fitting segmented curves whose joinpoints have to be estimated. *Journal of the American Statistical Association* 66:736–743.
- Kim, H.-Y., M. P. Fay, E. J. Feuer, and D. N. Midthune. 2000. Permutation methods for joinpoint regression with applications to cancer rates. *Statistics in Medicine* 19:335–351.
- Laidig, T. E., S. Ralston, and J. R. Bence. 1991. Dynamics of growth in the early life history of shortbelly rockfish, *Sebastes jordani*. U.S. National Marine Fisheries Service Fishery Bulletin 89:611–621.
- Lerman, P. M. 1980. Fitting segmented regression models by grid search. *Applied Statistics* 29:77–84.
- Maceina, M. J. 2007. Use of piecewise nonlinear models to estimate variable size-related mortality rates. *North American Journal of Fisheries Management* 27:971–977.
- McQuarrie, A. D., and C. L. Tsai. 1998. Regression and time series model selection. World Scientific Publishing, Singapore.
- National Cancer Institute. 2005. Joinpoint regression program, version 3.0. National Cancer Institute, Silver Spring, Maryland. Available: srab.cancer.gov/joinpoint/. (August 2007).
- Piepho, H. P., and J. O. Ogutu. 2003. Inference for the break point in segmented regression with application to longitudinal data. *Biometrical Journal* 45:591–601.
- SAS Institute. 2003. SAS/STAT user's guide. SAS Institute, Cary, North Carolina.
- SAS Institute. 2007. SAS/OR 9.1.3 user's guide: mathematical programming 3.1, volumes 1–4. SAS Institute, Cary, North Carolina.
- Seber, G. A. F., and C. J. Wild. 2003. Nonlinear regression. Wiley, Hoboken, New Jersey.
- Shuai, X., Z. Zhou, and R. S. Yost. 2003. Using segmented regression models to fit soil nutrient and soybean grain yield changes due to liming. *Journal of Agricultural, Biological, and Environmental Statistics* 8:240–252.
- Tiwari, R. C., K. A. Cronin, W. Davis, E. J. Feuer, B. Yu, and S. Chib. 2005. Bayesian model selection for joinpoint regression with application to age-adjusted cancer rates. *Applied Statistics* 54:919–939.
- Toms, J. D., and M. L. Lesperance. 2003. Piecewise regression: a tool for identifying ecological thresholds. *Ecology* 84:2034–2041.
- Zweifel, J. R., and R. Lasker. 1976. Prehatch and posthatch growth of fishes: a general model. U.S. National Marine Fisheries Service Fishery Bulletin 74:609–621.