STOCK ASSESSMENT MODELS

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Overview
We used age-structured population models in two ways. The first was as a means to generate estimates of lake trout and lake whitefish abundance and mortality rates and describe how these have changed over time. The second was to project yield, harvest amounts, and associated effort that met criteria established as part of the 2000 Consent Decree. The first of these tasks was accomplished through applying statistical catch-at-age analysis (SCAA) as a means of estimating parameters determining fish abundance and mortality. These catch-age models operated with annual time steps and age-specific abundances. Mortality rates were estimated for each year through the last year for which data were available. Models were developed for stocks in each defined management area.

The second task built from the first, by projecting the estimated fish population forward through the 2005 fishing season, accounting for expected fishing and natural mortality and projecting the associated harvest and yield. The fishing mortality rates were adjusted in these projections to match upper bounds on fishing effort, fishery harvest, or total mortality while satisfying state and tribal allocation as defined in the Consent Decree.

Statistical Catch-Age Analysis
A catch-age model was fit to available data. Each model consisted of two components. The first was a sub-model that described the population dynamics of the stock. The second was a sub-model that predicted observed data, given the estimated population each year. The agreement between the model predictions and observed data was measured by statistical likelihood. Both the population and observation sub-models included adjustable parameters. Any given set of these parameters corresponded to a specific sequence of stock abundances, mortality rates, and predicted data. The set of such parameters and associated stock dynamics and mortality rates that maximized the likelihood (the maximum likelihood estimates) was taken as the best estimate.

Population sub-model
The basic population model was quite simple. Except for the first age and first year, abundance-at-age at the start of each year was calculated recursively as the proportion of the cohort surviving from the start of the previous year:

\[ N_{a+1,y+1} = N_{a,y}P_{a,y} \]

The proportion surviving was modeled as

\[ P_{a,y} = e^{-Z_{a,y}} \]

where \( Z_{a,y} \) was the instantaneous mortality rate for age-\( a \) and year-\( y \). Total annual mortality (\( A=1-P \)) increases with increasing \( Z \), but asymptotes at 1.0. Mortality targets were usually expressed...
in terms of $A$, but could be expressed in terms of the equivalent $Z$.

A primary challenge in developing the stock assessment models was to break the total instantaneous mortality rate into components of interest that can be calculated from a suite of parameters, which can be estimated from available data. All the models include fishing mortality ($F$) and background natural mortality ($M$). All lake trout models and whitefish models for Lake Huron include sea lamprey induced mortality ($ML$). In addition, fishing mortality was usually broken into two subcomponents. Thus:

$$Z_{a,y} = F(1)_{a,y} + F(2)_{a,y} + M_a + ML_{a,y}$$

where $F(1)$ and $F(2)$ represent two fishery components (e.g., gill nets and trap nets, or sport and commercial). It was not possible to estimate all these rates as independent age- and year-specific components. To reduce the number of parameters, for each fishery component, the age- and year-specific fishing mortality rates are products of age-specific "selectivity" and year-specific "fishing intensity". In a purely separable model, selectivity was constant and thus each fishing mortality component was the product of an age ($S$) and year ($f$) effect:

$$F(i)_{a,y} = S(i)_{a} f(i)_{y}$$

In many of our assessment models we have relaxed the separability assumption, to account for changing selectivity resulting from changes in size-at-age, fishery behavior, or other causes. To do this we modeled the relationship between selectivity and age with a four-parameter double logistic function that provides a flexible dome-shaped relationship between selectivity and age, and includes asymptotic increases with age as a special case. When time-varying selectivity was desired, one of the parameters of this function (that controls selectivity for younger ages) was allowed to vary gradually over time, following a quadratic function in time. Thus, selectivity patterns over time were described by the three parameters of the quadratic function and the three other parameters of the logistic function.

Fishing intensity was the fishing mortality rate for ages that had a selectivity of 1.0. Fishing intensities were not estimated freely, but instead were assumed to be proportional to effort, up to a multiplicative deviation:

$$f(i) = q(i)E(i)ζ(i)$$

where $q$ was catchability (the proportionality constant), $E$ was observed effort, and $ζ$ was the deviation. During model fitting, large estimated deviations were penalized. However, in cases where fishery effort was not considered to be very informative regarding fishing mortality (generally for the lake trout models), this penalty was reduced to near zero making the procedure nearly identical to estimating the $f(i)$ directly.

The background natural mortality was assumed to be constant over time. For lake whitefish models and models of wild lake trout in Lake Superior, $M$ was assumed constant for all ages modeled, whereas for other lake trout models, $M$ was allowed to be higher for the younger ages. For the whitefish models $M$ was assumed known based on a published relationship between $M$ and growth model parameters and water temperature (Pauly 1980). For lake trout, while $M$
was estimated during model fitting, deviations from prior estimates, based on the same relationship used for whitefish, were penalized.

Sea lamprey mortality rates were not estimated during model fitting. Instead they were calculated based on observed wounding (sum of A1-A3 marks), as was done by Sitar et al. (1999). For a given size of lake trout, sea lamprey mortality was calculated by:

\[ ML = w \frac{(1 - p)}{p} \]

where \( w \) was the mean wounds per fish and \( p \) was an estimate of the probability of surviving an attack. Length-specific wounding rates were converted to age-specific rates using an age-length key.

**Lake Huron sea lamprey-induced mortality on lake whitefish**

In past stock assessments for Lake Huron lake whitefish, sea lamprey-induced mortality was calculated for specific length classes of whitefish in the spring, then an age-length distribution was applied to the length-specific mortality rates to estimate age-specific sea lamprey mortality of whitefish (Bence 2002). These age-specific mortality rates were assumed to be constant across years and constant across management units and input as data to the stock assessments in Lake Huron as a matrix of age- and year-specific sea lamprey mortality rates.

The method for calculating sea lamprey-induced mortality of whitefish in Lake Huron changed in the 2003 harvest limit year stock assessments. Marking rate data collected during August through December was used to estimate sea lamprey mortality, because the probability of survival used to estimate sea lamprey mortality of whitefish was collected during late summer and fall (Spangler et al. 1980). Age-specific marking rates for whitefish were estimated from year-specific marking rates and a long-term average marking rate in each management unit as:

\[ m_{a,t} = \frac{m_{a,y}}{1 - \left(\frac{m_{t} - m_{y}}{m_{t}}\right)} \]

where \( m \) is the average number of sea lamprey marks per fish, \( a \) is age class, \( t \) is year, and \( y \) is the time series under consideration. The time series varied somewhat by management unit but typically covered 1980-2003 in Lake Huron units. Essentially, the average marking rate on an age class was a function of the annual deviation in sea lamprey marking in a management from the long-term average marking rate in that unit and the average long-term marking rate on each age class. Sea lamprey-induced mortality was then calculated as in past years (Bence 2002) given a probability of survival of 0.25 from a sea lamprey attack.

In summary, 4 to 6 parameters were estimated during the fitting of the SCAA models to describe each fishery’s selectivity pattern, and a year-specific parameter was estimated associated with each fishery’s fishing intensity. We estimated from zero parameters (whitefish) up to two parameters (stocked lake trout) to describe background natural mortality. No additional parameters were estimated during model fitting to describe sea lamprey mortality, as these rates were calculated directly from wounding data.
In order to complete the population model and describe stock dynamics over time it was necessary to specify the initial numbers at age in the first year and the recruitment of the youngest age in each subsequent year. In the simplest cases each of these would be estimated as a free parameter during model fitting. We deviated from this simplest case in various ways. For stocked lake trout stocks, we modeled recruitment as the number of yearling equivalents actually stocked and calculated to move into an area (see Movement Matrices) multiplied by a year-specific "survival adjustment" factor. In this case the "survival adjustment" factors were estimated as parameters, with values deviating from 1.0 being penalized. Wild lake trout recruitment was modeled as a random walk function which was the product of the prior year's recruitment and a multiplicative deviation. The recruitment in the starting year of the model was estimated as a formal model parameter. Lake whitefish recruitment was estimated for each year, with deviations from recruitment expected based on a Ricker stock-recruit function (with parameters estimated during model fitting) being penalized. For stocked lake trout stocks, when age composition data was limited in earlier years, initial age compositions were based on the known number of lake trout that were stocked and a rough estimate of annual mortality, rather than being estimated during model fitting. For all the hatchery lake trout stocks, initial numbers for year classes known not to be stocked were set to zero.

**Movement Matrices and the calculation of yearling equivalents stocked**

Assessment models for lake trout on lakes Michigan and Huron were for hatchery-reared lake trout stocked into the lakes. The effective number of yearling lake trout stocked into a management unit was calculated as follows. First, we assumed that lake trout recruitment was based on stocked yearlings or fall fingerlings. The numbers of yearling equivalents were calculated as the number of yearlings stocked that year plus 0.40 times the number of fall fingerlings stocked the year before. Next the numbers stocked at various locations were adjusted for movement soon after stocking (before substantial spatially-varying mortality comes into play). This was done by apportioning fixed proportions of the numbers stocked at each location as being effectively stocked into each of the management areas (recruitment location) on the lake. These translations of numbers from stocking location to recruitment location were in the form of a "movement matrix." The numbers effectively stocked to a management unit (recruitment location) were then summed over the stocking locations. These effective numbers stocked were the input that was then adjusted upward or downward to account for year-specific variations (see above).

**The observation sub-model**

The observation sub-model predicts numbers of lake trout or lake whitefish killed by each fishing component by age. For the lake trout models survey catch per unit effort (CPUE) by age is also provided. Fishery kill was then converted into proportions-at-age and total number killed for comparison with data. Likewise, age-specific CPUE was converted into proportions-at-age and total CPUE for comparison with observed data.
Fishery kill was predicted using Baranov’s catch equation:

\[ C(i)a_y = \frac{F(i)a_y}{Z_a} N_a A(i)a_y \]

Note that no additional parameters not already needed for the population sub-model needed to be estimated.

Survey CPUE was predicted assuming proportionality between population abundance and expected CPUE, with selectivity following a logistic or double logistic function of age:

\[ CPUE_{a,y} = q(s)S(s)a_y N_{a,y} \]

where \( q(s) \) was survey catchability, and \( S(s) \) was survey selectivity. In some cases survey selectivity was allowed to vary over time in the same way as was fishery selectivity. The parameters of the survey selectivity function and survey catchability were new parameters that needed to be estimated which were not needed for the population sub-model.

**The Likelihood (defining the best fit)**

For numerical and coding reasons it was convenient to maximize the likelihood by minimizing the negative log likelihood. Let \( L \) stand for the total log-likelihood. This was calculated as the sum of a set of \( K \) independent components:

\[ L = L_1 + L_2 + L_3 + ... + L_K \]

Each component represents a data source or penalty, and the number of components varied among stocks and species. For each fishery that was included in the model there were three components: one for the total fishery kill each year, one for the fishery age composition each year, and one for the effort deviations for each year. These likelihood components were calculated under the assumption that total fishery kill and effort deviations were lognormal and that the proportions-at-age were determined by a multinomial distribution. When a survey was available, this provided two likelihood components: one for the total CPUE (lognormal) and one for the age composition (multinomial). An additional component came from variation about stock-recruit functions or numbers based on stocking. In the calculation of this penalty term, the deviations were treated as lognormal. When variation about a prior estimate of \( M \) was allowed, this contributed another term to the likelihood, and these variations were also assumed to be lognormal.

These various components were weighted by either the inverse of the variance associated with them (lognormal components) or the effective sample size (multinomial components). Here if \( X \) was lognormally distributed, variance refers to the variance of \( \ln(X) \). In the case of effort deviations, in those cases where effort was assumed to provide little information on fishing mortality these components were down-weighted by an arbitrarily small value. The square root of the log-scale variances for the lognormal variables was approximately equal to the coefficient of variation (\( CV \)) on the arithmetic scale. In the case of a multinomial variable:

\[ CV(p) = \sqrt{\frac{p(1-p)}{N}} \]

With these relationships in mind the modeling group considered information
on the likely measurement error associated with the various data sources and specified default variances for each type of data, which were adjusted in cases where additional information was available on data quality.

In the case of variations about recruitment expected based on either the stock-recruit function or the numbers stocked, an iterative approach was followed during model fitting. An initial value for the standard deviation for variations about expected values was specified and the model was fit. Then the standard deviation of the resulting deviations was calculated. The model was refit, adjusting the value of the input standard deviation until the deviation between the standard deviation value specified prior to model fitting and the value calculated after model fitting was minimized. A minimum deviation was defined when the ratio of pre-to post-standard deviation was closest to 1.0.

Calculation of Recommended Harvest Regulation Guidelines, Total Allowable Catch (TAC), and Total Allowable Effort (TAE)

In general, upper bound recommendations on yield and effort were calculated by first estimating population abundance-at-age at the start of the year and then adjusting fishing mortality either to meet mortality targets or to follow guidelines established in the Consent Decree for phasing in the targets. The resulting projection of yield or the effort associated with the fishing mortality then formed the basis of the recommendations.

We start by describing how we determined the maximum amount of yield that could be taken, consistent with a specific upper bound on total mortality. This was the procedure that underlies the modeling group's recommendations regarding harvest regulation guidelines, TACs, and TAEs. We then describe how the procedures were modified to account for specific details that only apply to some areas. For some areas these details include how the target mortality rates were "phased-in" as documented in the Consent Decree.

Target Mortality Rates

The Consent Decree specifies a "fully-phased in" upper bound target for total mortality (i.e., $A =$ the proportion of the population that dies in a year). These rates were either 40-45% (depending on area) for lake trout or 65% for lake whitefish. As demonstrated by the Interagency Modeling Group (IMG) during the period that the Consent Decree was negotiated, these target rates require additional structure in order to be uniquely defined. This occurs because mortality rates vary among ages, so whether or not a population was above a mortality target depends upon what ages were considered and how the mortality rates for the different ages were combined.

Following the procedure of the IMG, we uniquely define mortality rates by making use of the idea of spawning stock biomass per recruit (SSBR). For lake trout, we first calculate spawning stock biomass for a default target mortality schedule. Any age-specific mortality schedule that produces as much spawning stock biomass as the default schedule was considered to be at or below the target mortality rate. The default schedule was to have only natural mortality (excluding sea lamprey-induced mortality) for ages below a specified age, and mortality...
equal to the target rate for ages equal to or above the specified age. The specified age at which the target rate first applied varied among areas depending upon maturity schedules and precedent.

For whitefish a somewhat different procedure was used to ensure both that an adequate amount of spawning stock was achieved per recruit and that more than one age was contributing substantially to the spawning population. This was done following a two-stage approach. First, overall fishing mortality rates were adjusted so that during the projection period total annual mortality on the age experiencing the highest projected fishing mortality rate was equal to 65%. Then the spawning stock biomass per recruit was calculated for that scenario. Spawning potential reduction (SPR) was calculated by dividing this by the spawning stock biomass per recruit, calculated assuming only background natural mortality. If SPR was less than 0.2, fishing mortality was decreased until SPR was equal to 0.2. The approach was developed by examining various different "rules" and ascertaining that this approach generally ensured more than one age class was contributing substantially to spawning. A SPR of 0.2 was aggressive by standards applied in other fisheries and reflects a perception that lake whitefish was generally robust to fairly high fishing rates.

Population at the Start of the 2005 Fishing Year

The SCAA stock assessment models for lake trout directly estimate population abundance at the start of the year and mortality rates. As a result these estimates can be used in a straightforward fashion to project abundance for all ages other than the age of recruitment (the youngest age in the model) at the start of next year. Recruitment was set at a value reflecting recent levels of recruitment (Lake Superior) or expected stocking. Note that assumed recruitment has little influence on calculations of harvest during the next year, as these fish are either not selected or only weakly selected by the fishery.

Lake whitefish SCAA stock assessment models were similar to lake trout models except that the estimates were based on data two years behind the year for which a harvest limit was being calculated. Thus for lake whitefish there was one additional step, which was projecting the population for two years. For this projection, age-specific mortality rates by source (i.e., trap-net and gill-net fishing mortality, sea lamprey-induced mortality, natural mortality) were set equal to rates averaged over the last three years for which estimates were made. Recruitment of lake whitefish for the two projection years was set to the average recruitment during the last 10 years for which SCAA estimates were available.

Projections during the 2005 Fishing Season

Starting with the estimates or projections of age-specific abundance at the start of 2005, the population was projected forward over the year accounting for age-specific mortality rates by source, using the same equations described above for the SCAA models. Numbers harvested-at-age were calculated by application of the Baranov catch equation. Harvest-at-age was converted to yield by multiplying numbers harvested-at-age by weight-at-
age for the fishery and summing over ages.

In these calculations, background natural mortality ($M$) was left at the same value as was used or estimated in the SCAA assessments. Although this was calculated as the average rate in recent years in most of the projection sheets, currently $M$ was assumed constant over time in the assessment models. Likewise, sea lamprey-induced mortality was set to the average of the values in the last three years of the SCAA.

Fishing mortality rates by type (either sport and commercial or trap net and gill net for lake trout and lake whitefish, respectively) were based on average rates in recent years. These average rates were adjusted to account for changes stipulated in the Consent Decree or known changes in fishing activity by multiplying the baseline age-specific rates by an appropriate multiplier. For example, if a gill net fishery existed in an area prior to 2005, but did not in 2005, then in projecting whitefish yield the multiplier for gill-net fishery was set to zero. When fishing mortality was adjusted to account for a specified change in fishing effort, or when fishing effort was calculated to correspond with a specific level of fishing mortality rate, effort and fishing mortality were treated as being directly proportional. This basic approach to fishing mortality assumes that selectivity and catchability for each source will remain the same as it was on average in recent years. Detail on how fishing mortality rates were adjusted is covered in the next section.

Setting Fishing Mortality Rates for 2005

Fishing mortality rates were adjusted depending on specific details of how an area was designated in the Consent Decree. We begin by considering lake trout. The simplest case was for areas calculated under the assumption of no phase-in (also called ‘fully phased-in’ areas) and meeting Consent Decree mortality rate and allocation standards: MM-5, MM-67, MH-2, MI-5, MI-6, and MI-7. Additionally, MH-1 was considered partially phased-in. This was accomplished by setting the multipliers for the recreational and commercial fisheries so as to simultaneously meet the mortality target (expressed in terms of SSBR) and the designated allocation. The process of finding the correct multipliers was expedited by making use of the Solver utility within Microsoft Excel spreadsheets. In MM-5 the target mortality rate was 45% and the allocation was 60% state and 40% tribal. In MM-67 the target mortality rate was 40% and the allocation was 90% state, 10% tribal. In MH-1, the interim target mortality was 47%, and the allocation was 8% state and 92% tribal. In MH-2 the target mortality rate was 40% and the allocation was 95% state and 5% tribal. In MI-5 the target mortality rate was 45% and the allocation was 95% state and 5% tribal. In MI-6 the target mortality rate was 45% and the allocation was 50% state and 50% tribal. In MI-7, the target mortality rate was 45% and the allocation was 30% state and 70% tribal.

In the Lake Superior units adjustments were made as appropriate when reporting yield limits to account for the harvest of hatchery lake trout since tabled yield limits were taken as applying to all lean lake trout (wild and hatchery). This was necessary because hatchery lake trout, which were not part of the modeled population, do constitute a portion of the reported yields. The
recommended yield limits do not include siscowet lake trout. Sport fishery harvest was reported for lean lake trout. In MI-5, commercial yield was reported separately for lean lake trout. In MI-6 and MI-7 reported commercial yield included both lean and siscowet lake trout. The lean-siscowet composition was measured in commercial monitoring. Thus total yield can be 121% and 141% of the recommended yield limits for lean lake trout that we table. (Note that the harvest and survey data were adjusted so it reflected only lean, wild fish before they were compared with model predictions.)

The TAC for MM-4 was calculated under a phase-in of effort guidelines for commercial effort, recreational regulations, and associated harvest limits. The base period for commercial effort was 1997-1999. Hence we adjusted the average commercial fishing mortality rates during that period by multiplying them by the proportion of 1997-1999 large-mesh gill net effort that was remaining after conversion of gill net fishers to trap nets. Recreational effort was the average of 2002-2004 values, adjusted for any change in size limits. In 2003 the recreational size limit increased from 20” to 22”. Commercial TACs were based on predicted kill. The estimated allowable commercial yield was greater than the 20% change allowed in the Consent Decree, and the TFC agreed to accept the higher estimated TAC.

TAC calculations for MM-123 were more complicated than for other areas because of special provisions in the Consent Decree. Potential TACs were calculated two ways. First, TACs were calculated assuming that target mortality rates and allocation were fully phased in (40% mortality, allocation 10% state: 90% tribal). Second, TACs were calculated using a phase-in approach that is based on the previous years’ harvest, less the reduction in lake trout harvest projected from gill net reductions. Then, the larger of the tribal TACs among these two options was chosen. The state TAC was estimated as though the model were fully phased-in. Thus for the second option we followed the same approach as we used in other areas (i.e., based on 2002-2004 effort and any regulation changes). The phase-in approach was guided by the Consent Decree’s requirement that the tribal TAC be set to the 1997-1999 harvest adjusted for any change in effort. We did this by first calculating a 2005 yield based on no-conversion of gear (1997-1999 effort) and then calculating taking into account the proportion of large-mesh gill net that was converted (as for phase-in rules in other areas).

TAC estimates for fully phased-in units MM-5 and MM-6,7 were calculated as per the consent decree. The 2005 TACs for both management units decreased by more than 15% compared to the 2004 TACs. The TFC agreed to accept a higher estimated TAC for both units in 2005 limited by a 15% decline from the 2004 TAC’s for each unit.

Lake whitefish recommended yields were calculated generally following the approach used for fully phased-in lake trout areas. Details differed because of the different way that target mortality was defined for whitefish, and because for most areas there was no specified allocation between state and tribal fisheries (WFS-05 was an exception). In cases where there was no specified allocation, the first step was to adjust the multipliers for trap nets and gill nets to account for known changes in fishing...
effort (generally changes expected to arise from conversions or movement of operations). This step merely adjusts the relative contributions of the two gears. Then an overall multiplier (that applied to both gears) was adjusted until the target mortality rate was reached for the fully-selected age. When an allocation was specified the multipliers for the two gears were adjusted simultaneously (as was the case for lake trout) to match both mortality and allocation targets. At this point SPR was examined, and if it was below 0.20 the fishing multiplier was reduced until SPR reached 0.20.

References cited:


